Equivalence of Paths in Discrete Morse Theory

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Stratified Morse Theory generalizes Classical Morse Theory to spaces with singularities (e.g., real and complex analytic spaces). In this context, it is established a connection between the homological characterization of a stratified space X and the tangential and normal Morse data associated with each critical point of a Morse function $f : X \to \mathbb{R}$.

My research project, join work with Neza Mramor Kosta, Primoz Skraba and Gregor jerse, is aimed to generalize Discrete Morse Theory in this direction: we want to characterize discrete Morse functions on simplicial complexes that are triangulations of stratified topological spaces. In this seminar I'm going to show some results and, above all, the main ideas arisen from our several discussions on how to rightly answer to the following questions: Given a stratified simplicial complex and a gradient vector field on it, what are the tangential Morse data and the normal Morse data of a critical simplex? Moreover, how to provide a homological characterization of the space? Our approach is based on the analysis of the flow in the star and on the link of each simplex and on the concept of equivalent V-paths. The local behaviour of the function should provide information on the type of simplex (regular, critical, or singular), and on the stratum at which it belongs to. Moreover, the equivalence of V-paths should allow to pass from local to global information on homotopically trivial subcomplexes.