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Solution of Borsuk problem about special metrization of polyhedra and its applications

Abstract: Borsuk in his book "Theory of retracts" posed a problem (10.2): is it true that every polytope admits a localy strongly convex metric? (A metric d on a set M is locally strongly convex if every point of M has a neighborhood such that each point pair (x, y) in this neighborhood has precisely one centre z, i.e. $d(x, z) = d(z, y) = \frac{1}{2}d(x, y)$. The speaker shall give a talk about his solution (1983) of this problem and some applications of corresponding constructions of metric cones and suspensions of different curvatures over a metric space:

1) The construction of manifolds of dimension $n \geq 5$ with globally CAT(0)-metric which are not homeomorphic to $\mathbb{R}^n$ (giving a counterexample to Gromov conjecture) by Ancel and Giulbault (1997).

2) The construction of a $CAT(1)$-metric on arbitrary sphere $S^n, n \geq 5$ such that spaces of directions at some points are not homeomorphic to S^{n-1} by the speaker, which uses Edwards-Cannon theorem on double suspension.

3) A question of A.D.Aleksandrov and the speaker about corresponding metrics on 4-manifolds (1984), whose positive solution would imply a positive solution of Poincare conjecture (speaker, 2001).

In connection with his construction, the speaker will discuss the equivalence of Poincare conjecture to combinatorial character of every simplicial triangulation on a 4-manifold, discovered by Bing, and its connection with later results of Freedman and Donaldson about smooth structures on 4-manifolds. He also will discuss briefly a simplicial triangulation problem of topological manifolds in dimensions ≥ 5.