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On contractivity of multivalued mappings in the absence of the Hausdorff distance

Abstract: Basically a correlation between fixed points theories for singlevalued mappings and for multivalued mappings of metric spaces into themselves deals with the substitution of a given metric, say d, on a space X by the corresponding Hausdorff "metric" H_d on a set of all subsets of X. As a rule, in the case of singlevalued mappings a contractivity-like restriction of a type

$$d(f(x), f(y)) \leq \varphi(d(x, y)) < d(x, y) \qquad (*)$$

is replaced by its analog

$$H_d(F(x), F(y)) \leq \varphi(d(x, y)) < d(x, y). \qquad (**)$$

for a multivalued mapping F.

Recall that the inequality $H_d(A, B) < \varepsilon$ means that each of the sets A and B is a subset of an open $\varepsilon-$neighborhood of the other set. The main goal of the talk will be to show that the proximity of $F(x)$ and $F(y)$ with respect to H_d is in certain sense superfluous for successive approximations $x_n \to x_*$ tending to a fixed point of F, $x_* \in F(x_*)$.

Some introductory material and a discussion of some open problems in fixed point theory of multivalued mappings will be presented as well.