Complexity of graph manifolds

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The notion of complexity for compact 3-dimensional manifolds, was introduced by Matveev in the nineties as a way to measure how complicated a manifold is; indeed, for closed orientable irreducible manifolds, the complexity coincides with the minimum number of tetrahedra needed to construct the manifold, with the only exceptions of the 3-sphere, the projective space and the lens space L(3, 1) all having complexity zero. There exists a census of manifolds according to increasing complexity that, for the orientable case, is Recognizer catalogue and includes all manifolds up to complexity 12 (see `http://matlas.math.csu.ru/?page=search`).

In this talk, after recalling some general result about complexity, I will focus on an important class of manifolds: graph manifolds. These manifolds have been introduced and classified by Waldhausen in the sixties and are defined as compact 3-manifolds obtained by gluing Seifert fibre spaces along toric boundary components. I will present an upper bound for their complexity obtained in a joint work with Michele Mulazzani (University of Bologna) that is sharp for all the 14502 graph manifolds included in the Recognizer catalogue.