Simplicial complexes of polyomino tilings

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Polyomino shape is a union of unit squares connected edge by edge. In mathematical physics, they are also known as animals in the grid. We define a simplicial complex for a given set of polyomino shapes and a given subset of the square grid in a plane or on a torus. The topological and combinatorial properties of the complex are exciting. They reveal many features of placements of tiles from the given set into the given subset of the grid without overlappings. We will prove that these complexes have high connectivity and that, in some cases, they have the homotopy type of wedge of spheres. Some of their unexpected applications will be discussed.