Descriptive complexity in number theory and dynamics

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Informally, a real number is normal in base b if in its b-ary expansion, all digits and blocks of digits occur as often as one would expect them to, uniformly at random. Kechris asked several questions involving descriptive complexity of sets of normal numbers. The first of these was resolved in 1994 when Ki and Linton proved that the set of numbers normal in base b is Π_3^0-complete. Further questions were resolved by Becher and Slaman. Many of the techniques used in these proofs can be used elsewhere. We will discuss recent results where similar techniques were applied to solve a problem of Sharkovsky and Sivak and a question of Kolyada, Misiurewicz, and Snoha. Furthermore, we will discuss a recent result where the set of numbers that are continued fraction normal, but not normal in any base b, was shown to be complete at the expected level of $D_2(\Pi_3^0)$. An immediate corollary is that this set is uncountable, a result (due to Vandehey) only known previously assuming the generalized Riemann hypothesis.