Borsuk-Ulam property and sectional category

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For a Hausdorff space X, a free involution $\tau : X \to X$, and a Hausdorff space Y, we discover a connection between the sectional category of the double covers

$$q : X \to X/\tau \quad \text{and} \quad q_Y : F(Y,2) \to D(Y,2)$$

from the ordered configuration space $F(Y,2)$ to its unordered quotient $D(Y,2) = F(Y,2)/\Sigma_2$, and the Borsuk-Ulam property (BUP) for the triple $((X,\tau);Y)$. Explicitly, we demonstrate that the triple $((X,\tau);Y)$ satisfies the BUP if the sectional category of q is greater than the sectional category of q_Y.